UNIVERSAL MEAN VELOCITY PROFILE IN THE JET PART OF THE TURBULENT BOUNDARY LAYER

A. N. Boiko, L. N. Dymant, and V. M. Eroshenko

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The authors identify a jet region in the turbulent boundary layer and determine its mean velocity profile. This concept correlates a wide class of flows with different perturbation factors, including injection, longitudinal pressure gradient, and roughness.

In the present state of turbulent boundary layer theory it is important to construct correlations for the flow characteristics, in order to elucidate the physical laws of the flow and to use these correlations in various computation methods. The possibility of correlations rests on the conservative properties of the turbulent boundary layer. The stability of the laws of turbulence in the wall part of the layer is a premise of the theory of limiting transitions [1]. Moreover, the outer part of the layer adjacent to the unperturbed flow also possesses some conservation property.

The correlation in the present paper is based on the concept of the outer part of turbulent boundary layers having a jet-like nature, and these properties, similar to those of a free jet, are specific indications of the jet region and determine its size. Formal indications of this region of the boundary layer include a constant Prandtl mixing length, or a monotonic decrease in shear stress towards the outer edge of the boundary layer. It follows from physical arguments that the boundary between the wall region and the outer jet region in developed turbulent boundary layers will be a surface where the shear stress has a maximum [2]. This method of determining boundaries of the regions can be useful for turbulent boundary layers with arbitrary perturbations, including layers on an impermeable surface with zero pressure gradient. In the latter case transition to the jet region occurs where the shear stress begins to decrease appreciably. The validity of treating the flow in the outer region as a free turbulent boundary layer in a free jet is confirmed by the identical nature of the processes of entrainment of nonturbulent fluid from the external flow, the similarity of conditions at the outer boundary, the similar nature of conversion of energy of mean motion into turbulent energy, and by the similar behavior of the eddy viscosity. Finally, a confirmation of the ideas presented here concerning the jet nature of the flow in the outer part of turbulent boundary layers is the fact that there exists a universal dependence for velocities in variables which are similar to those used to describe universal velocity profiles in free jets and wakes.

We determine the form of the dependence by assuming a linear variation of the shear stress τ across the entire jet region. This hypothesis is based on numerous experimental and theoretical data for different kinds of boundary layers. It can be asserted quite confidently that a linear variation is typical for the jet region considered and that a certain amount of curvature of the profile at the boundaries of the region is apparently due to intermittency and to peculiarities of entrainment of nonturbulent fluid by large eddies.

From the Prandtl formula

$$\tau = \rho l^2 \left| \frac{\partial u}{\partial y} \right| \frac{\partial u}{\partial y} \quad (1)$$

allowing for the fact that the mixing length l is constant in the jet region, it follows that:

$$\frac{u_m - u}{u_m - u_{mm}} = A \left(\frac{y_m - y}{y_m - y_{mm}} \right)^{3/2}, \quad (2)$$

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where

$$A = \frac{2}{3l} \sqrt{\frac{\tau_m}{\rho}} \left(\frac{y_m - y_{mm}}{u_m - u_{mm}}\right). \tag{3}$$

Here the difference in the coordinates $y_m - y_{mm}$ is the thickness of the jet region, and the difference in the velocities $u_m - u_{mm}$ is the maximum velocity defect in the jet region. The coefficient A was determined from experimental data of different authors. It is noteworthy that for all types of boundary layers examined the coefficient A had the value unity. Comparison of Eq. (2) with A = 1 with test results is shown in Fig. 1a. It can be seen that the dependence, in contrast with earlier correlations considered, is universal in nature and gives a good description of the velocity distribution for a wide class of flows in the turbulent boundary layer. Figure 1a shows experimental data on velocity profiles in a channel with parallel walls for various values of Reynolds number [3], on a flat plate (the "one-seventh" law), on a permeable surface with a pressure gradient [4, 6] and without a pressure gradient [5], for various injection parameters, including strong blowing, where the profiles are S-shaped, in the wall jets on permeable surfaces in the presence of pressure gradient (the results of the present authors), and in a pipe with rough walls [7]. Equation (2) can also be written in velocity defect form. Since the mixing length is constant in the jet region, $l = \beta y_m$ [8], it follows from Eq. (3) that

$$\frac{u_m - u_{mm}}{\overset{*}{v}_m} = \beta'(1 - \eta_{mm}), \tag{4}$$

and then Eq. (2) takes the form

$$\frac{u_m - u}{\overset{*}{v}_m} = \frac{\beta'}{(1 - \eta_{mm})^{1/2}} (1 - \eta)^{3/2}. \tag{5}$$

Here the dynamic velocity $\overset{*}{v}_m$ is determined from the maximum shear stress, and is constant

$$\beta' = \frac{2}{3A\beta}.$$

It is worth noting that Eq. (5) is similar in form to the well-known Darcy formula [8], which was established experimentally for the special case of flow in circular channels:

$$\frac{u_m - u}{v^*} = 5.08 \left(\frac{R - y}{R}\right)^{3/2}. \tag{6}$$

The left sides of the two equations are identical, since the shear stress τ_m coincides with τ_w for flow in pipe flow, and the coordinate η_{mm} = const.

A specific confirmation of the validity of Eq. (2) with the value A = 1 is that it correlates with the approved relations for special type of flow. For example, from the Darcy formula, after it is converted to the form

$$\frac{u_m - u}{u_m - u_{mm}} \cdot \frac{u_m - u_{mm}}{v^*} = 5.08 \left(\frac{R - y_{mm}}{R}\right)^{3/2} \left(\frac{R - y}{R - y_{mm}}\right)^{3/2} \tag{7}$$

allowing for the fact that R = y_m and that Eq. (7) also holds for y = y_{mm}, it follows that

$$\frac{u_m - u}{u_m - u_{mm}} = \left(\frac{y_m - y}{y_m - y_{mm}}\right)^{3/2}. \tag{8}$$

It is known that the Darcy formula corresponds to the actual velocity distribution for $y/R \geq 0.25$, which corresponds to the jet region of the boundary layer as defined above.

Similarly one can correlate Eq. (2) with the Ross relation for flow with pressure gradient [9]

$$\frac{u}{u_m} = 1 - D\left(1 - \frac{y}{y_m}\right)^{3/2}, \tag{9}$$

where D is a shape factor determined by the flow conditions. From Eq. (8), after conversion to the form

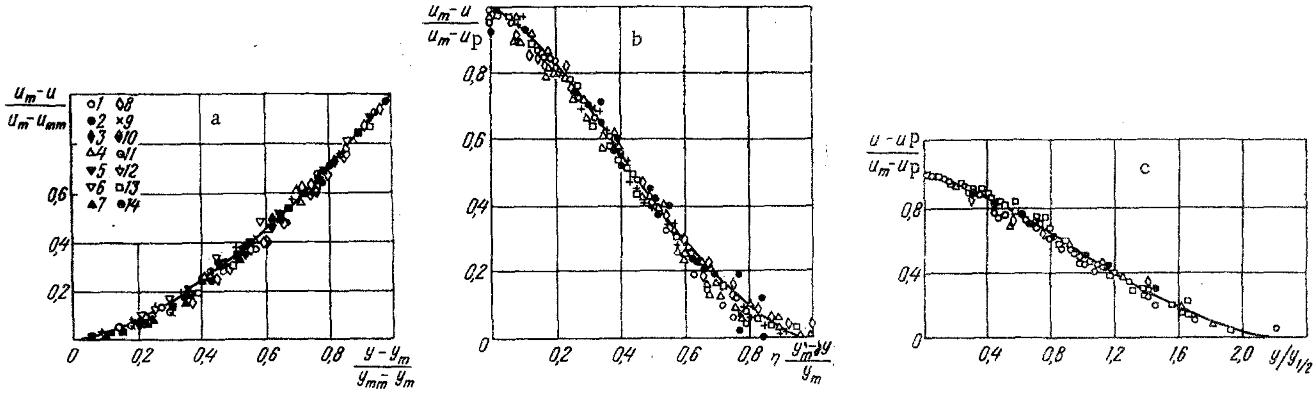

Fig. 1. Comparison of the universal correlation (2) with experimental data for wall turbulent flow (a); for a boundary layer in the initial section of a free jet (b), and for the main part of a free jet (c): 1) Re = 57,000, [3]; 2) Re = 230,000, [3]; 3) m = 0, [5]; 4) m = 0.005, [5]; 5) m = 0.01, [5]; 6) m = 0.018, [5]; 7) m = 0.0212, [5]; 8) m = 0.0147, [4]; 9) the "one-seventh" law; 10) m = 0.016, $\bar{x}$ = 6.19, [6]; 11) m = 0.048, $\bar{x}$ = 6.19, [6]; 12) Re = $4 \cdot 10^4$, $\bar{x}$ = 30, [7]; 13) wall jet on a permeable surface, m = 0.03, r = 0.5; 14) wall jet on a permeable surface, m = 0.03, r = 2.

$$\frac{u_m - u}{u_m - u_{mm}} \frac{u_m - u_{mm}}{u_m} = D\left(\frac{y_m - y_{mm}}{y_m}\right)^{3/2}\left(\frac{y_m - y}{y_m - y_{mm}}\right)^{3/2} \tag{10}$$

one obtains Eq. (2) with A = 1.

One can check the concept that the flow in the outer part of the boundary layer is jet-like in nature by comparing the proposed universal correlation with data obtained in free jets. Figure 1b compares the universal correlation with experimental results for the boundary layer in the initial section of a jet, as given in [10]. The upper part of the graph corresponds to the outer edge of the jet boundary layer, and the lower branch corresponds to its internal region. In the coordinates of the graphs Eq. (2) takes the form, for the outer and inner regions, respectively:

$$\frac{u_m - u}{u_m - u_p} = 1 - \frac{7}{16}\left(\frac{y_2 - y}{0.391}\right)^{3/2} \tag{11}$$

and

$$\frac{u_m - u}{u_m - u_p} = \frac{9}{16}\left[\left(1 - \frac{y_2 - y}{\delta}\right)\frac{1}{0.604}\right]^{3/2}. \tag{12}$$

The coordinate y_{mm} in the jet boundary layer is determined as the coordinate of the point where $\partial^2 u/\partial y^2 = 0$ (which corresponds to τ_m for l = const), and here one uses the velocity profile in the Schlichting form

$$\frac{u_m - u}{u_m - u_p} = \left[1 - \left(\frac{y - y_2}{\delta}\right)^{3/2}\right]^2. \tag{13}$$

Figure 1c compares Eq. (2) with experimental data for the main part of a free jet [10]. The upper branch of the curve corresponds to the central part of the jet, and the lower part corresponds to the outer part.

The satisfactory agreement between the universal correlations (2) and the experimental data on free jets confirms that the present correlation is physically well founded.

In conclusion attention should be drawn to one of the consequences of the ideas presented here concerning characteristic regions of the turbulent boundary layer. This is that an increase in the intensity of blowing through a permeable surface, and also an increase in a positive pressure gradient do not cause conditions in the boundary layer to approach the conditions in a free jet, as is widely assumed, but on the contrary, cause the jet region to contract, as is evidenced by the displacement of the shear stress maximum away from the wall. This fact should be taken into account in the development of computational methods for boundary layers.

NOTATION

y	is the vertical coordinate;
δ	is the boundary layer thickness;
η	is the dimensionless transverse coordinate;
$y_{0.5}$	is the vertical coordinate of the point where $u = 0.5\ u_m$;
y_2	is the distance from the outer edge of the layer;
l	is the mixing length;
u	is the longitudinal mean velocity;
τ	is the shear stress;
v^*	is the dynamic velocity;
ρ	is the density;
R	is the tube radius;
m	is the intensity of transverse blowing;
u_0	is the initial velocity of wall jet;
$r = u_m / u_0$	is the wall jet parallelism parameter;
x	is the longitudinal coordinate;
$\bar{x} = x / \delta$	is the dimensionless longitudinal coordinate.

Subscripts

m	denotes the maximum;
mm	denotes on the line of maximum shear stress;
w	denotes at the wall;
p	denotes the parallel flow.

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